Occurence of Characteristic Peaks in Phononic Multilayer Structures

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In this paper the acoustic transmission properties of multilayer structures was analyzed. Were compared binary and aperiodic (Severin, Thue-Morse) superlattices, Calculations were performed using the Transfer Matrix Method (TMM) algorithm. As a superlattice environment in the simulation the water was used. The material used to construct the structure was a PNM-0.38PT piezoelectric. Multilayer types have been selected so that the total number of layers for a given generation is equal in all structures.

Keywords: phononic, transfer marix, aperiodic, multilayers, acoustic

Intensive research on photonic [1-8] and phononic [9, 10] structures, especially their transmission properties, are being conducted. The properties of these structures are less dependent on the materials used, but much more on the geometrical distribution of the materials [11-16]. Modern production capabilities of composite materials allow for the design of structures with very high accuracy [17, 18]. This made it possible to manufacture materials for photonics and phononics characterized by the presence of band gaps. Created multilayers can be used as filters, waveguides or for demultiplexing of acoustic waves [19-25].

TMM model

The transmission was determined using the Transfer Matrix Method (TMM) algorithm [26]. The pressure p in the multilayer medium (where v_i is phase velocity of the acoustic wave in i layer and t is time) can be described as

$$\frac{1}{v_i^2}\frac{\partial^2 p}{\partial^2 t^2} - \nabla^2 p = 0 \tag{1}$$

For a 1D case, the solution takes the form

$$p_i = P_i(x)e^{-i\omega t} \tag{2}$$

where

$$P_i(x) = A_i e^{ik_i x} + B_i e^{-ik_i x} \tag{3}$$

$$k_i = \frac{\omega}{v_i} = \frac{2\pi f}{v_i} \tag{4}$$

A_i and B_i coefficients in terms of eq. (3) represents respectively the forward and the reflect wave in *i* layer. Frequency is denoted as *f*.

The characteristic matrix Γ of the TMM model for the superlattice structure can be described by

$$\Gamma = \Xi_{in,1}\left(\prod_{i=1}^{n-1}\Psi_i \Xi_{i,i+1}\right)\Psi_n \Xi_{n,out} \tag{5}$$

The propagation matrix ψ_i in layer *i* with thickness d_i is defined as

$$\Psi_i = \begin{bmatrix} e^{i\frac{2\pi f d_i}{v_i}} & 0 \\ 0 & e^{-i\frac{2\pi f d_i}{v_i}} \end{bmatrix} \tag{6}$$

The transfer matrix $\Xi_{i,j}$ where ρ_i is mass density of layer i is represent by

$$\Xi_{i,j} = \frac{1}{2}\begin{bmatrix} \frac{\rho_i v_i + \rho_j v_j}{\rho_i v_i} & \frac{\rho_i v_i - \rho_j v_j}{\rho_i v_i} \\ \frac{\rho_i v_i + \rho_j v_j}{\rho_i v_i} & \frac{\rho_i v_i + \rho_j v_j}{\rho_i v_i} \end{bmatrix} \tag{7}$$

From the characteristic matrix (5) the transmission can be determined by

$$T = \left|\Gamma_{1,1}\right|^{-2} \tag{8}$$

Analyzed structures

Structures types were selected so that for a given generation number *L* they had the same number of layers. Analyzed distribution of layers in structures for generation numbers *L* from 2 to 4 is shown in table 1.

Binary

Concatenation rule for periodic binary structure X_L^B creation is given by

$$X_L^B = (AB)^{2^{L-1}} \tag{9}$$

An exponent 2^{L-1} means a repeat of the AB cell sequence for *L* generation number.

Severin

The aperiodic Severin structure X_L^S can be obtained using the recursive rule of substitution

$$\begin{cases} A \rightarrow BB \\ B \rightarrow AB \end{cases} \tag{10}$$

Where the initial condition is

$$X_0^S = B \tag{11}$$

Thue-Morse

The recursive rule of substitution for aperiodic Thue-Morse structure $X_L^{T\text{-}M}$ can be obtained using the

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$$\begin{cases} A \rightarrow AB \\ B \rightarrow BA \end{cases} \quad (12)$$

The initial condition for Thue-Morse structure is

$$X_0^{T-M} = A \quad (13)$$

Research

The structures were analyzed in the frequency range up to 500 kHz. Multilayers was surrounded by water which was material A with phase velocity 1500 m/s and mass density 1000 kg/m^3. Material B was 0.2 mol% Fe-doped PMN-0.38PT [0 0 1] with mass density 8093 kg/m^3 and phase velocity 4410 m/s. Thickness of layers in the structures was 5 mm. Figures 1, 2 and 3 show the transmission calculated by TMM model for periodic binary and aperiodic Severin and Thue-Morse structures respectively for *L* generation number from 2 to 5.

In figure 1-3, it can be seen that with the increase in the L value the complexity of the transmission band structure increases. From the calculated transmission data for each structure, the values of the first derivative were estimated using difference approximations. Then, for the values of the first derivative near the zeroes, a linear approximation was made which allowed to determine the zero of the first derivative and at the same time determine the maxima of the transmission function.

When analyzing table 2, it should be noted that some transmission bands occur in two or even three structures studied. The 6 kHz and 453 kHz bands are present in all investigated structures for the L = 4 generation number. Bands 149, 299, 301 and 453 kHz are found in all structures for L = 4 and in some for L = 3. The most interesting is the 441 kHz band, which despite the strongly different construction of multilayer systems occurs for all analyzed cases. The remaining bands (marked in light gray in table 2) are present in only two structures.

L	X_L^B	X_L^S	X_L^{T-M}
2	ABAB	BBAB	ABBA
3	ABABABAB	ABABBBAB	ABBABAAB
4	ABABABABABABABAB	BBABBBABABABBBAB	ABBABAABBAABABBA
5	ABABABABABABABAB ABABABABABABABAB	ABABBBABABABBBAB BBABBBABABABBBAB	ABBABAABBAABABBA BAABABBAABBABAAB

Table 1
ARRANGEMENT OF THE LAYERS OF ANALYZED STRUCTURES FOR *L* GENERATIONS NUMBER

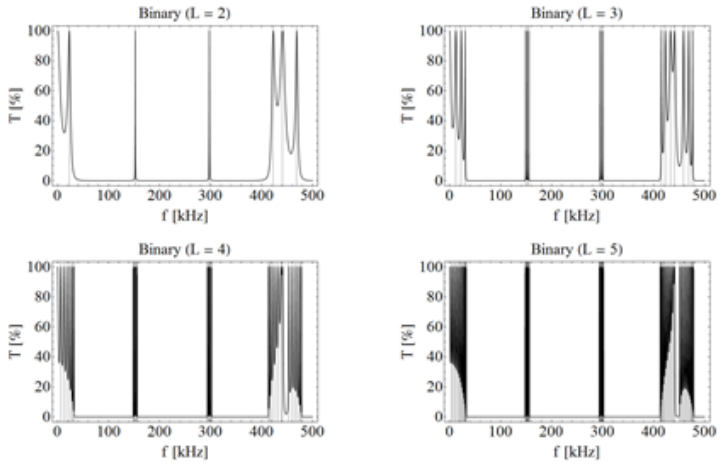

Fig. 1. Transmission maps of binary superlattices calculated by TMM algorithm for *L* generations

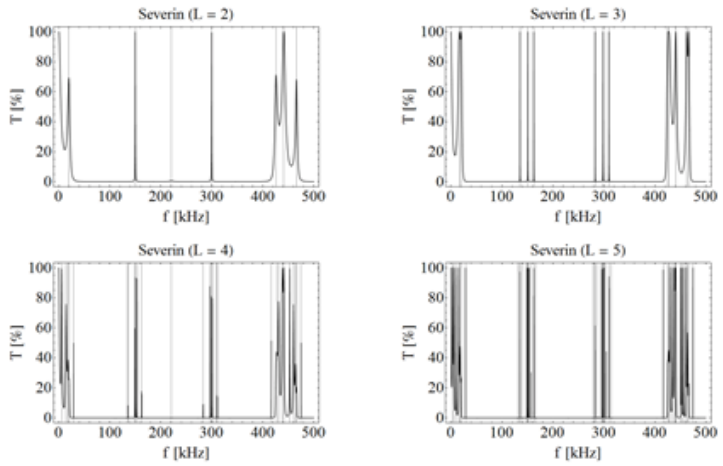

Fig. 2. Transmission maps of Severin superlattices calculated by TMM algorithm for *L* generations

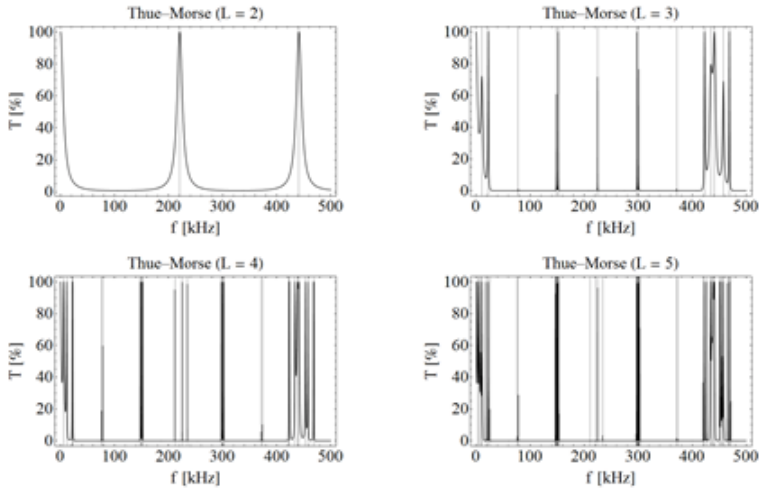

Fig. 3. Transmission maps of Thue-Morse superlattices calculated by TMM algorithm for *L* generations

Table 2

FREQUENCIES IN kHZ FOR WHICH THE TRANSMISSION VALUES OF THE STUDIED STRUCTURES REACH THEIR MAXIMUM. LIGHT GRAY COLOR HIGHLIGHTS THE VALUES OCCURRING IN THE TWO STRUCTURES. DARK GRAY INDICATES VALUES IN ALL THREE TYPES OF STRUCTURES

L.p.	X_L^B			X_L^S			X_L^{T-M}		
	L = 2	L = 3	L = 4	L = 2	L = 3	L = 4	L = 2	L = 3	L = 4
1			6			6			6
2		12	12			14		11	11
3			18		17	18			22
4	23	23	23	20	20	20		23	23
5			27			29			76
6		30	30			135		78	78
7			32		136	136			148
8			148			149		149	149
9		149	149	150	151	151			151
10			150			153		152	152
11	152	152	152			162			211
12			154		163	163	220	225	225
13		155	155	221		221			234
14			156		283	283		298	298
15			293			283			299
16		294	294			296		301	301
17			296		299	299			302
18	298	298	298	300		301		371	371
19			299		310	310			374
20		301	301			311		423	423
21			302			417			424
22			413	426	426	427		435	434
23		415	415		428	431			439
24			419			438	441	441	441
25	423	423	423	441	441	441			453
26			428			453		458	458
27		433	433			460			468
28			439		463	464		469	469
29	441	441	441	466	467	466			
30			453			475			
31		459	459						
32			464						
33	469	469	469						
34			473						
35		476	476						
36			478						

Conclusions

The study examined the transmission bands for the three different but constructed of the same materials structures. The superlattices were selected so that their lengths were the same for different *L* generation numbers. They were made of the same materials and with the same thickness of layer. It has been noted that, despite a very different internal structure, some transmission bands were present for all analyzed structures. On the other hand, some were found in only two structures. It would be important to conduct a broader study to determine what factors influence the occurrence of certain bands in all structures.

References

1.SZOTA M., NABIALEK M., GARUS S., GARUS J., K. BLOCH, Archives of Materials Science and Engineering **64/2**, 2013, pp. 213-218

2.GARUS J., GARUS S., GRUSZKA K., BLOCH K., NABIALEK M., In¿ynieria Materialowa **2** (**198**), 2014, pp. 113-116

3.GARUS J., GARUS S., SZOTA M., NABIALEK M., GRUSZKA K., Archives of Materials Science and Engineering **64/1**, 2013, pp. 20-27

4.GARUS S., GRUSZKA K., GARUS J., BLOCH K., NABIALEK M., DOSPIAL, M., SZOTA M., In¿ynieria Materialowa **2 (198)**, 2014, pp. 117-120

5.GARUS S., GARUS J., SZOTA M., NABIALEK M., GRUSZKA K., BLOCH K., Journal of Achievements in Materials and Manufacturing Engineering **61/2**, 2013, pp. 327-335

6.GARUS S., GARUS J., SZLAZAK K., NABIALEK M., PIETRUSIEWICZ P., B£OCH K., GRUSZKA K., SZOTA M., Journal of Achievements in Materials and Manufacturing Engineering **61/2**, 2013, pp. 236-243

7.GARUS J., GARUS S., B£OCH K., SZOTA M., NABIALEK M., SZLAZAK K., Journal of Achievements in Materials and Manufacturing Engineering **61/2**, 2013, pp. 229-235

8.GARUS S., GARUS J., SZOTA M., NABIALEK M., GRUSZKA K., BLOCH K., Archives of Materials Science and Engineering **64/2**, 2013, pp. 110-117

9.GRUSZKA K., GARUS S., GARUS J., BLOCH K., NABIALEK M., In¿ynieria Materialowa **2 (198)**, 2014, pp. 132-135

10.GRUSZKA K., GARUS S., NABIALEK M., BLOCH K., GONDRO J., SZOTA M., PAJAK B., Journal of Achievements in Materials and Manufacturing Engineering **61/2**, 2013, pp. 250-256

11.GRUSZKA K., NABIALEK M., SZOTA M., Archives of Materials Science and Engineering **66(2)**, 2014, pp. 74-80

12.GRUSZKA K., NABIALEK M., SZOTA M., Archives of Materials Science and Engineering **68(1)**, 2014, pp.24-30

13.RADU, C.D., PARTENI, O., SANDU, I.G., LISA, G., MUNTEANU, C., LUPU, V.V., Rev. Chim. (Bucharest), **67**, no. 1, 2016, p. 199.

14. TUGUI, C.A., NEJNERU, C., GALUSCA, D.G., PERJU, M.C., AXINTE M., CIMPOESU N., VIZUREANU P., Journal of Optoelectronics and Advanced Materials, **17**, no. 11-12, 2015, p. 1855.

15.ACHITEI, D.C., VIZUREANU, P., DANA, D., CIMPOESU, N., Metalurgia International, **18**, SI2, 2013, p. 104.

16.PETRISOR, S.M., BARSAN, G., Proceedings of SPIE, **9067**, 2013, article 90671M

17.GAWDZINSKA K., BRYLL K., NAGOLSKA D., Arch. Metall. Mater., **61 (1)**, 2016, pp. 177–182

18.GAWDZIÑSKA K., CHYBOWSKI L., BEJGER A., KRILE S., Metalurgija, **55 (4)**, 2016, pp. 659-662

19.QIU C. Y., LIU Z. Y., JUN Z. M., SHI J., Applied Physics Letters **87**, 2005, pp. 104101

20.CICEK A., KAYA O. A., YILMAZ M., ULUG B., Journal of Applied Physics **111**, 2012, pp. 013522

21.ZHANG M. D., ZHONG W., ZHANG X. D., Journal of Applied Physics **111**, 2012, pp. 104314

22.SANCHEZ-DEHESA J., GARCIA-CHOCANO V. M., TORRENT D., CERVERA F., Cabrera S., The Journal of the Acoustical Society of America **129**, 2011, pp. 1173

23.WU T. T., WU L. C., HUANG Z. G., Journal of Applied Physics **97**, 2005, pp. 094916

24.PENNEC Y., DJAFARI-ROUHANI B., VASSEUR J. O., KHELIF A., DEYMIER P. A., Physical Review E - Statistical, Nonlinear, and Soft Matter Physics **69**, 2004, pp. 046608

25.OLSSON R. H., EL-KADY I., Measurement Science and Technology **20**, 2009, pp. 012002

26.KRIEGEL I., SCOTOGNELLA F., In Physica E: Low-dimensional Systems and Nanostructures **85**, 2017, pp. 34-37

Manuscript received: 12.11.2017